Keeping it R.E.A.L.

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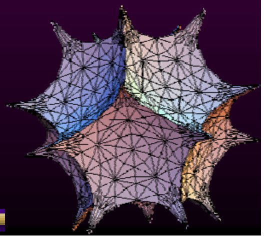

NSF-UBM

- Multiple team-taught courses per year
  - Mathematical Models in Biology
  - Biometry
  - Biomechanics/Biophysics
- Two semester ‘Calculus with Functions I,II’ that is equivalent to Calculus I (whatever equivalent means)
- Interdisciplinary research with biologists

James Madison University's Computational Courses

- Math 248 – Computers and Numerical Algorithms
  - Programming in a high-level language
  - Introduction to algorithms involving rootfinding, solving systems of linear equations, integrating, differentiating, and interpolating.

James Madison University's Computational Courses

- Math 448 – Numerical Analysis
  - Further study and analysis of algorithms used to solve nonlinear equations, systems of linear and nonlinear equations. Includes iterative methods.
- Math 449 – Numerical Analysis for Differential Equations
  - Further study and analysis of numerical techniques to solve ordinary and partial differential equations.

Keeping it R.E.A.L.

Keeping it R.E.A.L.: Research Experiences for All Learners

Research and Classroom Projects in Computational Mathematics

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MOSAIC emphases

- Algorithmic development
  - Converting words to a written algorithm (mathematical expression)
  - Converting the written algorithm basic code to perform the ‘experiment’
- Computers aren’t perfect!!!!!
  - Examples that show that computers don’t actually know everything and we are just trying to pry something out of them

Project #1

- Consider an iterated function that takes as input a number between zero and one and outputs either

  i) a number that is twice as big if the original result is less than one

  OR

  ii) a number that is twice as big minus one if the original result is greater than one.

  The output value becomes the input value for the next iteration.

Bernoulli Map

- Also known as the dyadic transformation or 2n mod 1 problem:

$$x_{n+1} = \begin{cases} 2x_n, & \text{if } 2x_n < 1 \\ 2x_n - 1, & \text{if } 2x_n \geq 1 \end{cases}$$

- Treat as a fixed point iteration with initial condition x_0.

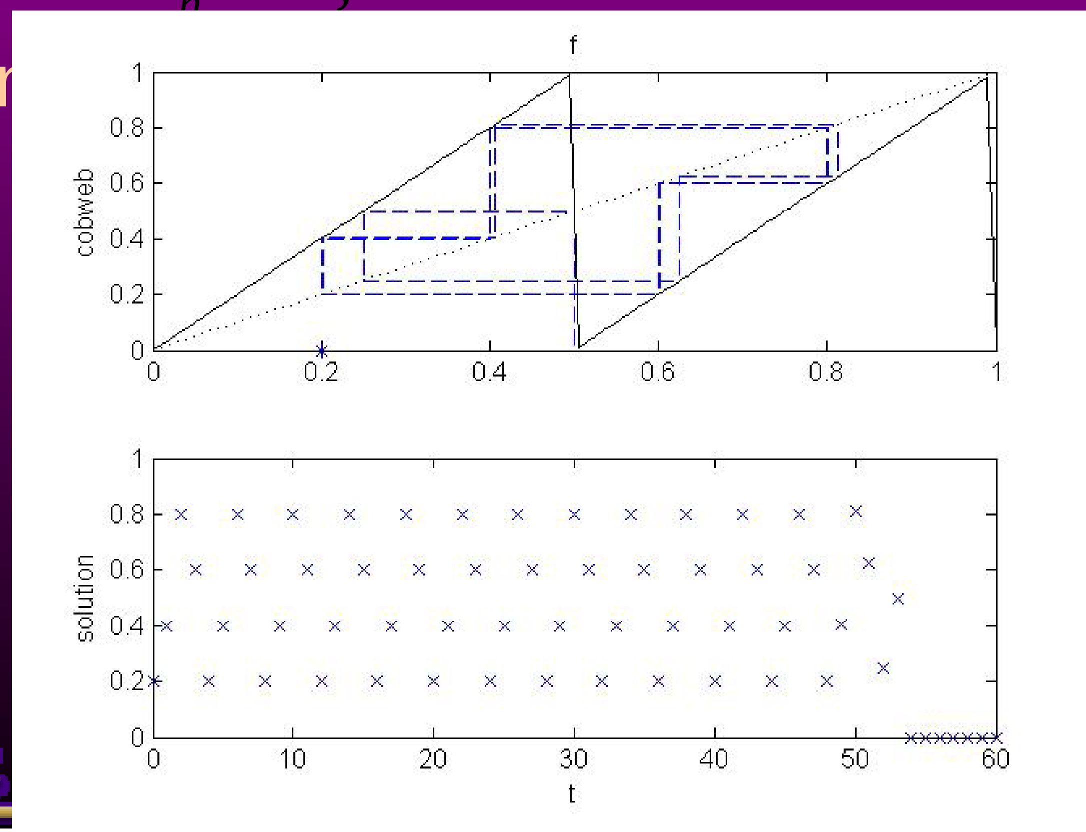
Bernoulli Map

- Find the initial value(s) that never changes when plugged into the Bernoulli map.
- Find period two and period four orbits. What determines the period of the orbit?
- Given *n*, can you always find a period *n* orbit? If so, what is it?
- Introduce cobweb diagram and terminology of fixed point iteration

Fixed Point Iterations

$$x_{n-1} = \begin{cases} 2x_n, & \text{if } 2x_n < 1 \\ 2x_n - 1, & \text{if } 2x_n \geq 1 \end{cases}$$

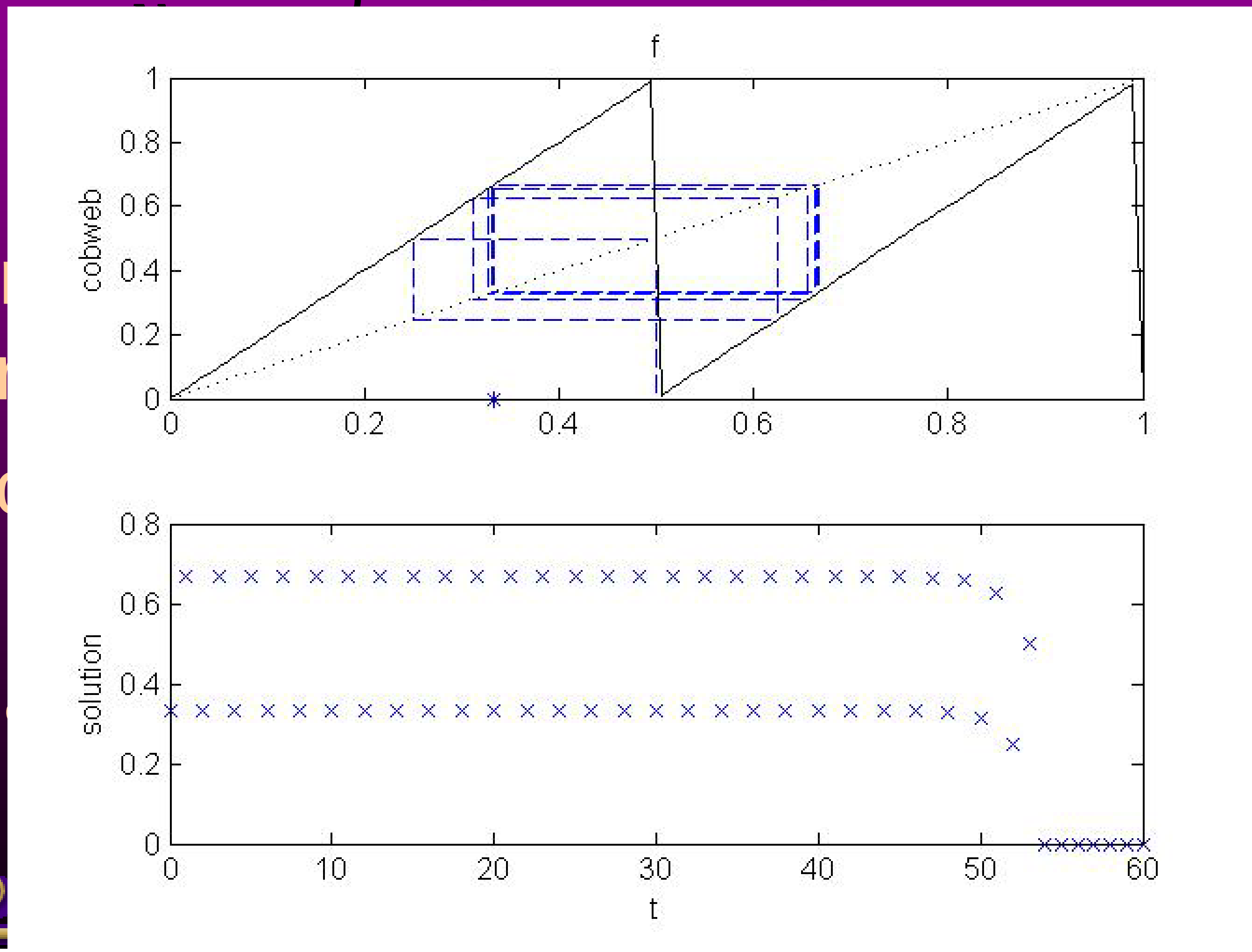
- Find fixed point(s) $\quad x_n = 0,1$
- Fixed point iteration
- Cobweb diagram
- Periodic orbits



Computational Bernoulli Map

$$\begin{cases} 2x_n, & \text{if } 2x_n < 1 \end{cases}$$

- Usi… e
  corr…
- It do… ey
  are …
- Wh…



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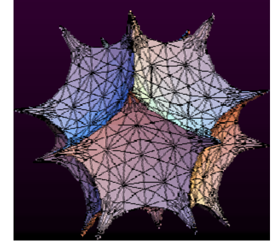
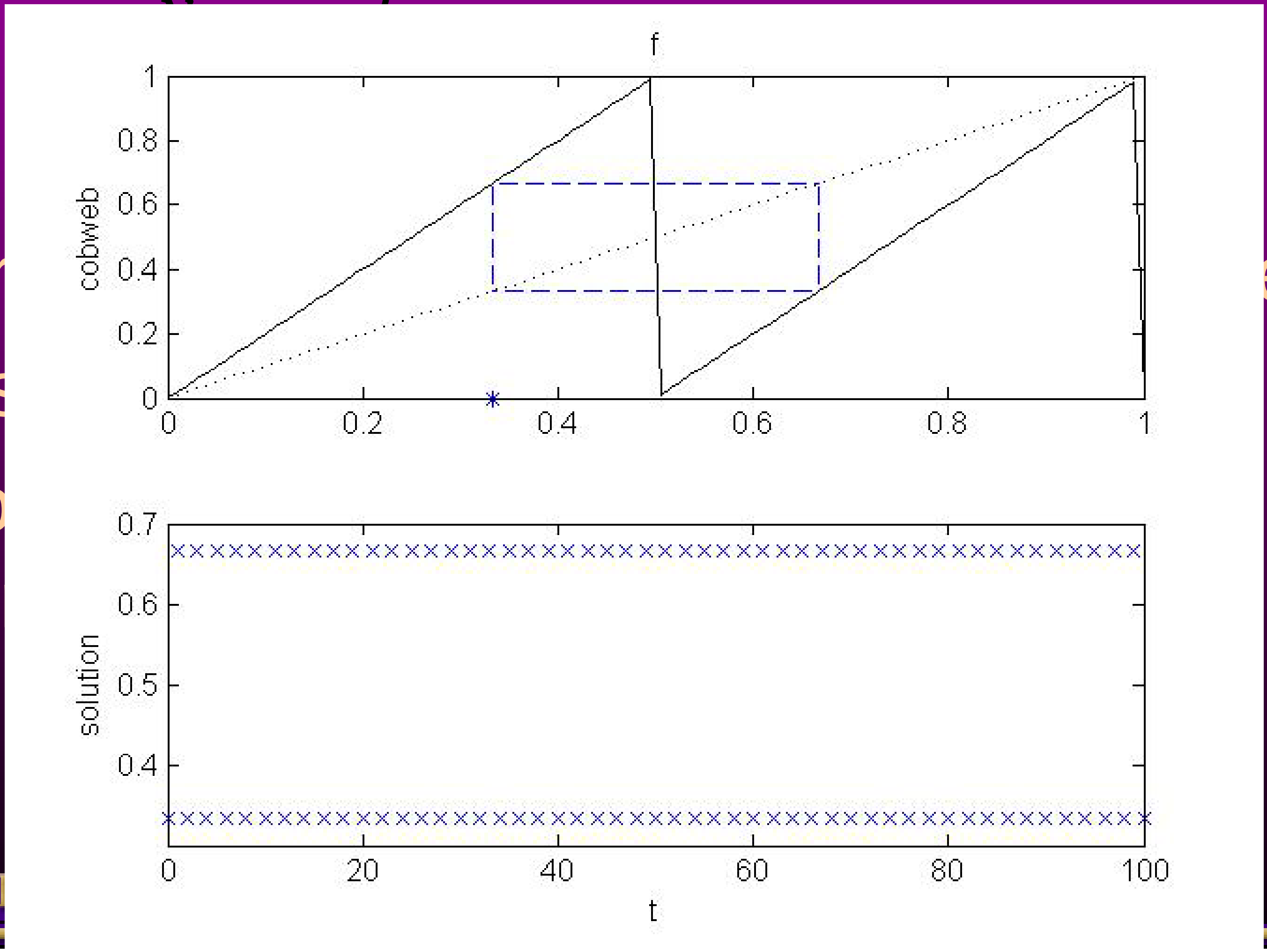


Computational Bernoulli Map

$$\begin{cases} 2x_n, & \text{if } 2x_n < 1 \end{cases}$$



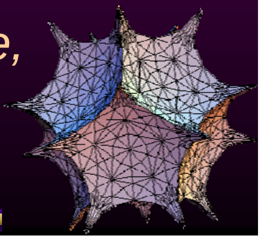
- Wr ... ers
- Dis
- Do
  nu

Conway's Prime Producing Machine*

$$\begin{array}{cccccccccccccc} \frac{17}{91}, & \frac{78}{85}, & \frac{19}{51}, & \frac{23}{38}, & \frac{29}{33}, & \frac{77}{29}, & \frac{95}{23}, & \frac{77}{19}, & \frac{1}{17}, & \frac{11}{13}, & \frac{13}{11}, & \frac{15}{14}, & \frac{15}{2}, & \frac{55}{1} \\ A & B & D & H & E & F & I & R & P & S & T & L & M & N \end{array}$$

- The input is 2.
- A step involves multiplying the current number by the leftmost member of the above table which gives a whole number answer.
- Output happens whenever a pure power of 2 occurs.

**Guy, RK, "Conway's Prime Producing Machine", Mathematics Magazine, 56:1, 26-33, 1983.*

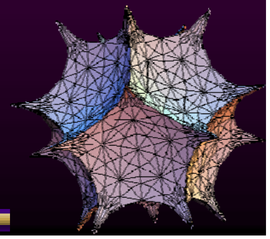


Conway’s Prime Producing Machine

$$\frac{17}{91}, \frac{78}{85}, \frac{19}{51}, \frac{23}{38}, \frac{29}{33}, \frac{77}{29}, \frac{95}{23}, \frac{77}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{15}{14}, \frac{15}{2}, \frac{55}{1}$$

A	B	D	H	E	F	I	R	P	S	T	L	M	N

2		A	425	B	156
M	15	B	390	S	132
N	825	S	330	E	116
E	725	E	290	F	308
F	1925	F	770	T	364
T	2275	T	910	A	68
		A	170	P	4



Conway's Prime Producing Machine

- Horribly inefficient
- Importance of using types
  - Using type double without any modifications, the students cannot produce the first prime number
  - Using int isn't too much better
- Impact of roundoff error

A Rather Normal Project

- A normal number is a number whose digits are equally distributed.
- What is an example of a normal number?

Normal number facts

- The set of normal numbers is dense in the real numbers.
- Easier to find non-normal numbers; focus on values between zero and one.
- Every rational number is eventually periodic and therefore not normal.
- Two irrational numbers have been proven to be normal: Champernowne's number and the Copeland-Erdos constant.

Last example

- Go to town!

$$x_{n+1} = \begin{cases} x_n / 2, & \text{if } x_n \text{ is even} \\ 3x_n + 1, & \text{if } x_n \text{ is odd} \end{cases}$$

Other possibilities

- Chaotic Rabbits
- Fractions of fractions
- Diffusion by chance
- Follow the rules!!
- Getting to the root of the problem

Conclusions

- The previous examples allow the students to go from concept to algorithm to code
- Conway's Prime Producing Machine and the Bernoulli Map are two examples that help students better understand how computers store real values and effectively demonstrate roundoff error.
- Fun – depending on how you define fun!

Thanks

- Dr. Carla Martin
- Numerous Math 248 students and future Math 235 students

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